Advanced Algorithm

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Lecture 7: Introduction of Approximation Algorithm

- Ref: Approximation Algorithm - Vijay V. Vazirani
- Decision problem: YES or No answer
- Optimization problem
- Approximation ratio

- General graph $G = (V, E)$, find the cardinality of maximum matching
- Polynomial time exact algorithm: very complicated
    - https://en.wikipedia.org/wiki/Blossom_algorithm
- Approximation algorithm: find a maximal matching
    - $1/2$-approximation ratio
    - The analysis is tight.

- Min, Max problem
- Analysis of a particular approximation algorithm
    - Approximation ratio: $\frac{\text{Algorithm's solution}}{\text{Optimal solution}}$ in the worst case
    - Is the analysis tight?
- Hardness of Approximation

Cardinality Vertex Cover

- Ref: Approximation Algorithm, Chapter 1.1
- Given graph $G = (V, E)$, find a set $S \subseteq V$ with minimum cardinality, s.t. $\forall (i, j) \in E, i \in S$ or $j \in S$.
- NP-complete
- Approximation algorithm: maximal matching
  - Analysis: 2-approximation ratio, tight

Set Cover

- Ref: Approximation Algorithm, Chapter 2
- Given a universal set $U = \{u_1, \cdots, u_n\}$, m subset $S_1, \cdots, S_m \subseteq U$ and a weight function $c : \{1, \cdots, m\} \to \mathbb{N}$. Find some sets which can cover all elements with minimum cost.
- Vertex cover problem is a special case of set cover problem
- Other examples: dominating set problem, edge cover problem
- Algorithm 1: greedy algorithm
    - Analysis: $O(\log n)$-approximation ratio, tight
- Algorithm 2: layering technique
    - Analysis: f-approximation ratio, tight (f is the frequency)
- 2-approximation algorithm for weighted vertex cover

- VC is APX-complete: cannot be approximated arbitrarily well unless $P = NP$
- VC cannot be approximated within a factor of 1.3606 unless $P = NP$ (2005, PCP)
- VC cannot be approximated within a factor of $2 - \epsilon$ if unique game conjecture is true (2008)
- Other assumption:
  - ETH (exponential time hypothesis): 3-SAT cannot be solved in subexponential time in the worst case

- Approximation Algorithm - Homework 1.1, 1.3, Page 7